



Theoretical modeling and experimental realization of dynamically magnified thermoacoustic-piezoelectric energy harvesters



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ABSTRACT

Conventional thermoacoustic-piezoelectric (TAP) harvesters convert thermal energy, such as solar or waste heat energy, directly into electrical energy without the need for any moving components. The input thermal energy generates a steep temperature gradient along a porous medium. At a critical threshold of the temperature gradient, self-sustained acoustic waves are developed inside an acoustic resonator. The associated pressure fluctuations impinge on a piezoelectric diaphragm, placed at the end of the resonator. In this study, the TAP harvester is coupled with an auxiliary elastic structure in the form of a simple spring-mass system to amplify the strain experienced by the piezoelectric element. The auxiliary structure is referred to as a dynamic magnifier and has been shown in different areas to significantly amplify the deflection of vibrating structures. A comprehensive model of the dynamically magnified thermoacoustic-piezoelectric (DMTAP) harvester has been developed that includes equations of motions of the system's mechanical components, the harvested voltage, the mechanical impedance of the coupled structure at the resonator end and the equations necessary to compute the self-excited frequencies of oscillations inside the acoustic resonator. Theoretical results confirmed that significant amplification of the harvested power is feasible if the magnifier's parameters are properly chosen. The performance characteristics of experimental prototypes of a thermoacoustic-piezoelectric resonator with and without the magnifier are examined. The obtained experimental findings are validated against the theoretical results. Dynamic magnifiers serve as a novel approach to enhance the effectiveness of thermoacoustic energy harvested from waste heat by increasing the efficiency of their harvesting components.

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1. Introduction

Thermoacoustics is an emergent technology that involves interaction of sound fields with solid surfaces to develop heat engines or refrigerators [1]. In a typical thermoacoustic engine, acoustically excited gas parcels can experience a piston-free thermodynamic cycle. In the presence of a temperature gradient that exceeds a specific threshold, an acoustic wave is

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sustained while the gas parcels are kept in intimate thermal contact with the adjacent solid boundaries. In a standing wave thermoacoustic engine, the solid surface is referred to as the stack. The stack is placed between two heat exchangers that act as a heat source and a sink, and the three components are placed inside an acoustic cavity called the resonator. The pressure and velocity fluctuations in the stack are such that heat is given to the oscillating gas particles at high pressures while it is removed at lower pressures so as to satisfy Rayleigh's criterion [1].

Considerable attention has been devoted to the development of such a class of engines in recent years [2–4]. This attention can be attributed to the fact that such devices are compact and possess few, if any, moving parts. Thermoacoustic engines are environmentally benign and can be driven by any source of heat, waste heat [5] or concentrated solar power [6,7]. The induced acoustic wave carries energy that can be usefully converted into electric power by means of a linear alternator or piezoelectric membranes, thus creating a viable alternative class of energy harvesters.

The integration of thermoacoustic systems with piezoelectric elements has recently been investigated. Keolian and Bastyr [8] placed emphasis on the development of large scale engines with a proposed system that involved heavy moving masses communicating with arrays of piezoelectric alternators. Symko et al. [9,10] presented primarily experimental work focusing on small scale devices for thermal management of microelectronics. Wekin [11] and Matveev et al. [12] introduced a simplified mathematical model to explain the operation of a standing wave thermoacoustic engine coupled with a piezoelectric transducer, referred to as the thermoacoustic-piezoelectric (*TAP*) energy harvester. In a *TAP* harvester, a piezoelectric transducer is placed at the end of the resonator to harness power from the incoming acoustic waves, generated in the stack, and convert it into electricity. The model enables the prediction of the device's operating frequency and onset temperature difference as a function of the characteristics of the piezoelectric element. Smoker et al. [13] and Zhao et al. [14–16] presented a detailed performance analysis of an experimental prototype of a *TAP* energy harvester.

Optimization of the stack parameters such as the material, porosity, spacing and/or the resonator geometry have been shown to magnify the amount of acoustic energy generated in the stack, thus ultimately improving the thermoacoustic device's overall efficiency. However, very few attempts have been devoted towards the improvement of a harvester component. Nough et al. [17,18] have presented a radically different approach whereby a conventional *TAP* harvester is coupled with an elastic structure in the form of a simple spring–mass system to amplify the strain experienced by the piezoelectric element. The proposed system is referred to as a dynamic magnifier and has been shown in different areas to significantly amplify the deflection of vibrating structures [19–21]. Dynamically magnified thermoacoustic-piezoelectric systems (*DMTAP*) can be advantageous when the appropriate properties of the magnifier are chosen. The *DMTAP* can be designed to achieve a higher efficiency than a conventional *TAP* of the same size, a higher voltage output and/or a lower temperature gradient across the stack ends. In previous attempts to discuss thermoacoustic harvesters with coupled dynamic magnifiers, the analysis was always limited to theoretical predictions extracted from a mathematical model of the coupled system. The present work attempts to show the potential and feasibility of *DMTAP* energy harvesters experimentally as the means of improving the output power from thermoacoustic-piezoelectric harvesters. Experimentally developed prototypes of this class of energy harvesters are tested and validated against mathematical predictions. The current work also confirms that self-sustained thermoacoustically induced oscillations inside the harvester's resonator are not jeopardized by the introduction of the elastic coupling, provided proper selection of the magnifier's parameters is done.

The paper is organized in 4 sections. Following the brief introduction outlined in Section 1, a quick mathematical overview of the equations governing thermoacoustic-piezoelectric harvesters with and without dynamic magnifiers is presented in Section 2. In Section 3, the experimental performance of the *DMTAP* in comparison with a conventional *TAP* harvester is discussed. The conclusions are summarized in Section 4.

2. Mathematical problem and governing equations

2.1. Pressure and velocity

The behavior of thermoacoustic devices is generally described by a one-dimensional linear mathematical model. Such a simplified model is usually attributed to the fact that the device dimensions in the direction of the working gas displacement are greater enough than those normal to it to ignore possible nonlinear effects. It is, however, worth noting here that more complex models of thermoacoustic devices that take into account nonlinearities arising from the response of such systems to cross-sectional area variations, stack position, gas properties and other design variables have been introduced in the literature [22–24]. For simplicity, the mathematical overview given here will be restricted to the simplified linear behavior.

For sinusoidal one-dimensional acoustic wave propagation in a resonator in the x -direction, the spatial components of pressure $P(x)$ and velocity $u(x)$ at any point x along the resonator are coupled by a set of two differential equations:

$$\frac{d^2 P(x)}{dx^2} + k^2 P(x) = 0 \quad (1)$$

$$u(x) = \frac{i}{\rho\omega} \frac{dP(x)}{dx} \quad (2)$$

where ρ is the density of the working gas. The wavenumber k is a complex expression, the real part of which is (ω/c) where ω is the oscillation's angular frequency and c is the speed of sound in the working gas, and its imaginary part is α denoting the wave's attenuation factor. Assuming the stack's length does not significantly influence the standing wave inside the resonator (short stack approximation, [2]), the pressure and velocity waveforms can be described by

$$P(x) = (A_1 e^{-ikx} + A_2 e^{ikx}) \quad (3)$$

and

$$u(x) = \frac{1}{\rho c} (A_1 e^{-ikx} + A_2 e^{ikx}) \quad (4)$$

where A_1 and A_2 are amplitude constants which are equal for a resonator that is closed at $x=0$, such as in *TAP* systems. The specific acoustic impedance at $x=L$ is given by

$$Z(L) = \frac{P(L)}{u(L)} \quad (5)$$

2.2. Oscillation frequency

In the *TAP* device shown in Fig. 1, a piezoelectric element of cross section A_p , mass m , damping coefficient b , and stiffness s is connected to the end of the tube at $x=L$. Assuming the piezoelectric element oscillates with one degree of freedom x_2 in the x -direction for simplicity [17], the two equations governing its oscillation are given by

$$m\ddot{x}_2 + b\dot{x}_2 + sx_2 - d_{33}sV_2 - \frac{A_R P(L)}{k_s} = 0 \quad (6)$$

and

$$sd_{33}\dot{x}_2 + C_p \dot{V}_2 + \frac{V_2}{Z_L} = 0 \quad (7)$$

where d_{33} and C_p are the piezoelectric strain coefficient and clamped capacitance in the poling x -direction. Also, V_2 denotes the voltage across the load Z_L , A_R defines the Helmholtz resonator's cross section area and k_s is A_R/A_p .

Eqs. (6) and (7) can be solved simultaneously to obtain an expression for $P(L)/\dot{x}_2$ representing the equivalent specific impedance of the piezoelectric element. This expression is then equated with $Z(L) = P(L)/u(L)$ in order to ensure pressure and velocity continuation at the interface between the piezoelectric element and the working gas at the end of the resonator. This gives

$$f_{\text{TAP}}(\omega, \alpha) = 0 \quad (8)$$

where f_{TAP} is given by

$$f_{\text{TAP}} = i(\rho c) \cot\left(\left[\frac{\omega}{c} + \alpha i\right]L\right) - \frac{k_s}{A_R} \left[i\omega m + b + \frac{s}{i\omega} + \frac{(d_{33}s)^2 Z_L}{1 + i\omega Z_L C_p} \right] \quad (9)$$

The real and imaginary parts of Eq. (8) are equal to zero and can therefore be used to generate two simultaneous equations, solving which gives the complex wavenumber and hence the oscillating frequency of the *TAP* harvester.

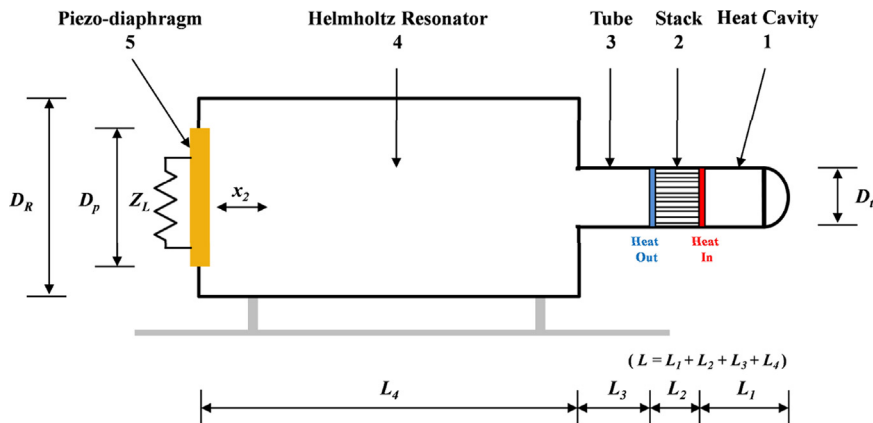


Fig. 1. Schematic drawing of a standing wave thermoacoustic-piezoelectric engine (*TAP*).

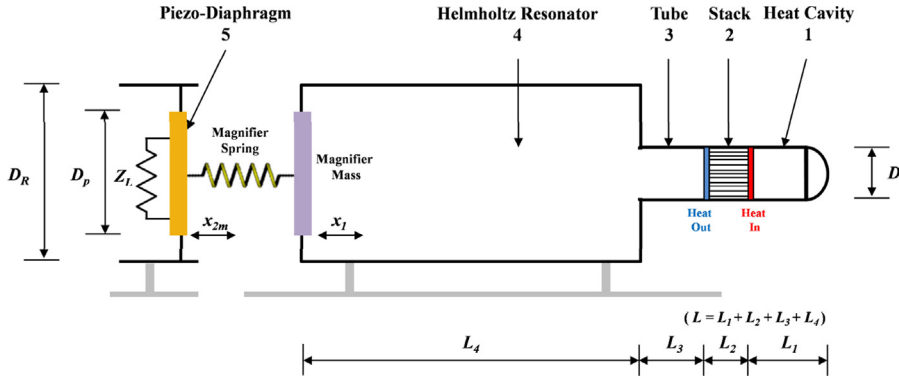


Fig. 2. Schematic drawing of a standing wave thermoacoustic-piezoelectric engine integrated with a dynamic magnifier (DMTAP).

For a DMTAP harvester shown in Fig. 2, the added parameters are m_m , c_m and k_m representing the mass, damping and stiffness of the added mass, respectively, as well as k_c denoting the stiffness of the spring connecting the mass and the piezoelectric element. The equations governing the system's three degrees of freedom (x_1 , x_{2m} and V_{2m}) are

$$m_m \ddot{x}_1 + c_m \dot{x}_1 + k_m x_1 + k_c (x_1 - x_{2m}) - \frac{A_R P(L)}{k_s} = 0 \quad (10)$$

$$m \ddot{x}_{2m} + b \dot{x}_{2m} + s x_{2m} + k_c (x_{2m} - x_1) - d_{33} s V_{2m} = 0 \quad (11)$$

and

$$s d_{33} \dot{x}_{2m} + C_p \dot{V}_{2m} + \frac{V_{2m}}{Z_L} = 0 \quad (12)$$

Solving Eqs. (10)–(12) simultaneously yields the equivalent impedance of a magnifier system. Equating the magnifier's impedance with $Z(L)$ gives

$$f_{\text{DMTAP}}(\omega, \alpha) = 0 \quad (13)$$

where f_{DMTAP} is given by

$$f_{\text{DMTAP}} = i(\rho c) \cot\left(\left[\frac{\omega}{c} + ai\right]L\right) - \frac{k_s [i\omega m_m + c_m + (k_c/i\omega) + (k_m/i\omega)][i\omega m + b + (s/i\omega) + (k_c/i\omega) + (\psi^2 Z_L / (1 + i\omega Z_L C_p))]] + (k_c^2 / \omega^2)}{[i\omega m + b + (s/i\omega) + (k_c/i\omega) + (\psi^2 Z_L / (1 + i\omega Z_L C_p))]} \quad (14)$$

Solving Eq. (13) for both real and imaginary parts yields the frequency of the self-excited oscillations of the DMTAP system and the corresponding attenuation factor. Substituting those back in the waveform equations gives the pressure and velocity at any point x along the resonator. The velocity of the gas particles at $x = L$ should match that of the piezoelectric element $\dot{x}_2$ in the TAP case, and that of the magnifier mass $\dot{x}_1$ in the DMTAP case. Eqs. (7) and (12) can then be used to estimate the output piezoelectric voltage. Achieving a V_{2m}/V_2 ratio greater than unity should indicate that the energy output from the DMTAP harvester is higher than that obtained from the conventional TAP harvester. Proper selection of the different magnifier parameters is critical to achieving this condition. Design optimization of the DMTAP harvester to satisfy different design objectives is presented in detail by Nouh et al. [18].

3. Experimental work

3.1. Speaker-driven harvester with a dynamic magnifier (no stack)

An experimental investigation of the performance of the DMTAP discussed earlier is presented. We start with a simpler setup than the design suggested by Fig. 2. The acoustic oscillations produced by the stack are simulated by a speaker placed at the beginning of the resonator. Since the harvesters are driven by speakers, we can drive the system at any desired frequency. The resonator is 2.75 in. (6.985 cm) in diameter. A circular buzzer PZT-4 piezoelectric element placed on a 0.008 in. (0.203 mm) thick aluminum sheet of a diameter equal to that of the tube is attached to the other end of the resonator and is then connected to a similar one through a mechanical spring of known stiffness. These piezoelectric elements, manufactured by Digi-Key, are light, inexpensive and more practical to use in experiments where multiple configurations are to be tested. Since several dynamic magnifiers were attempted (corresponding to different combinations of mass and stiffness ratios), these PZT-4 buzzers were optimal for this study. However, they produce much less voltage for the same amplitude of impinging acoustic waves when compared to the PZT-5A diaphragms used in similar analyses [13], making the overall efficiency of the harvesters presented here reasonably lower (Fig. 3).

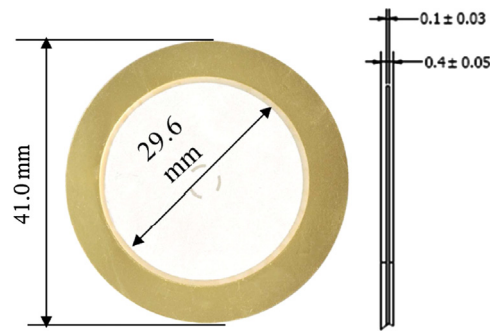


Fig. 3. Digi-key 41 mm Buzzer Piezoelectric element.

The second piezoelectric element is supported by a separate stand from the rest of the resonator. Furthermore, the section between the two piezoelectric elements is open to air. This way, by detaching the connecting spring and the second piezoelectric element, the system at hand is simply a speaker-driven cavity with one piezoelectric element at one end. The piezoelectric element converts the incoming acoustic energy from the speaker into an electrical output, thus simulating a *TAP*-like system. When the second piezoelectric element is reattached to the first one using the coupling spring, the system in effect simulates a *DMTAP*-like system. The first piezoelectric element in this case acts as the dynamic magnifier mass. For simplicity, the two systems at hand will be referred to as *TAP* and *DMTAP* in this section. A schematic of the experimental setup is shown in Fig. 4.

3.1.1. Voltage output from piezoelectric diaphragms

For different values of the stiffness k_c of the spring connecting the two piezoelectric elements, the system is found to have a first natural frequency in the range starting from 400 to 600 Hz when operating as a *TAP*- or a *DMTAP*-like system. Thus, using the speaker as the source of the acoustic energy and with no electrical loads on the piezoelectric elements, a sine sweep over the domain 0–800 Hz is carried out while monitoring the voltage output from both piezoelectric elements (V_1 and V_{2m}) over this range of frequencies. Having values of V_{2m} less than V_1 would mean that the *TAP* still operates as a better energy harvester than the *DMTAP*. This would eliminate the need for having to compare V_{2m} with the voltage V_2 from the first piezoelectric element when no springs are attached (*i.e.* the *TAP* case). However, having values of V_{2m} higher than V_1 is an indication that the power output in the second piezoelectric element is more than that obtained from the first one. Even though this can be taken as a valid approximation for the magnification in most cases, the voltage output from the second piezoelectric element V_{2m} should still be compared with the voltage V_2 to confirm that the proposed system does serve as a magnifier of the power harvested.

The variable parameters in such an experiment are mainly the spring constant k_c and the masses of the 2 piezoelectric elements. Both piezoelectric elements are similar and have the same weight. Both elements are supported by an aluminum backing of the same thickness, thus their total masses are equal. Small masses in the order of 1–5 g can still be attached to the piezoelectric elements as a way of varying the mass of the *DMTAP*. Several combinations of k_c and added weights are attempted while monitoring the voltage output over the sine swept frequency range on a signal analyzer. Fig. 5 shows an example of an undesirable scenario where the energy traveling from the first to the second piezoelectric element through the coupling spring is absorbed rather than magnified; hence V_1 values are higher than V_{2m} over the considered frequency range. The spring used here has a constant of k_c equal to 17,800 N/m. In this case, the added spring–mass structure serves as the means of dissipating the energy being conveyed to the second piezoelectric element instead of amplifying it. The figure displays also the nature of the dominant modes in the frequency spectrum. Notable are the two modes, at 490 and 580 Hz, resulting from combining the harvester with the dynamic magnifier. However, because of the weak nature of the coupling between the harvester and the magnifier, the contribution of the second mode to the output voltage is also weak.

Upon the addition of small masses to the piezoelectric elements, the voltage obtained from the second element begins improving in comparison with the case with no masses added. However, it is the combination of the masses and the proper spring constant that decides the performance of the *DMTAP*. The best results are obtained using a spring of k_c equal to around 29,180 N/m with no added masses. In this case, a strong coupling exists between the piezoelectric element and the dynamic magnifier resulting in enhanced performance.

Fig. 6 shows the response of the voltages: V_1 , V_2 and V_{2m} for this case. Note that V_1 in this case rises from its peak of 70 mV, in Fig. 5, to about 160 mV, while V_{2m} dramatically jumps from a peak of 8 mV to almost 450 mV. It is evident here that the energy transferred through the spring to the second piezoelectric element is magnified. This is manifested clearly by comparing the performance with that of the case without a magnifier where the output voltage V_2 peaks at about 290 mV and hence is much lower than the 450 mV achieved by the second element in the *DMTAP* case.

The brief mathematical model presented earlier is used to model the system at hand and compare with the experimental results. A *MATLAB* code is developed to simulate the experiment and predict values of V_1 , V_1 and V_{2m} over the frequency range 0–800 Hz for the cases presented in Fig. 6. Eqs. (9) and (14) are obtained for a resonator with a rigid end at the hot

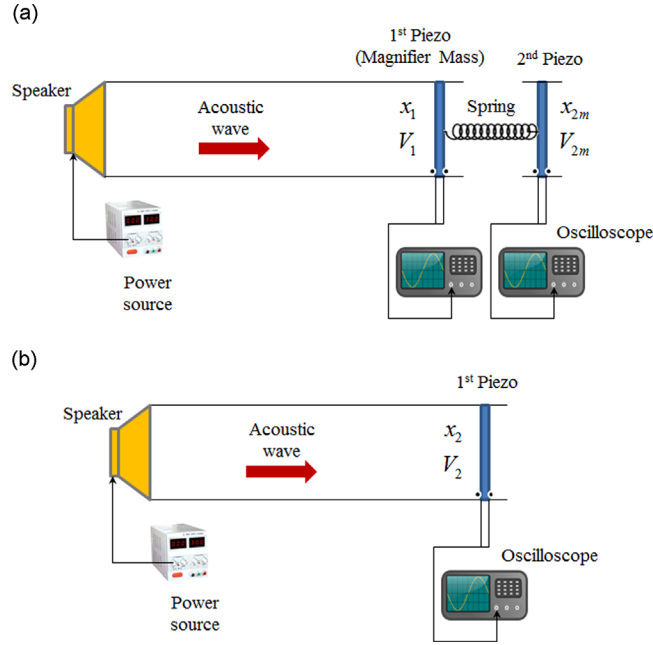


Fig. 4. Schematic diagram of the experimental setup when used as (a) DMTAP-like system and (b) TAP-like system.

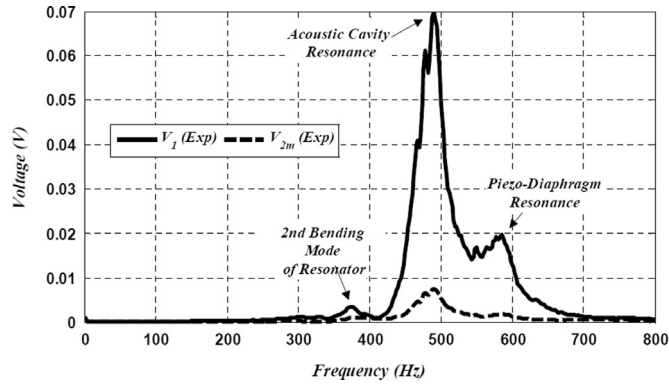


Fig. 5. Sine swept frequency response of voltage outputs V_1 and V_{2m} of the two piezoelectric elements ($k_c = 17,800$ N/m).

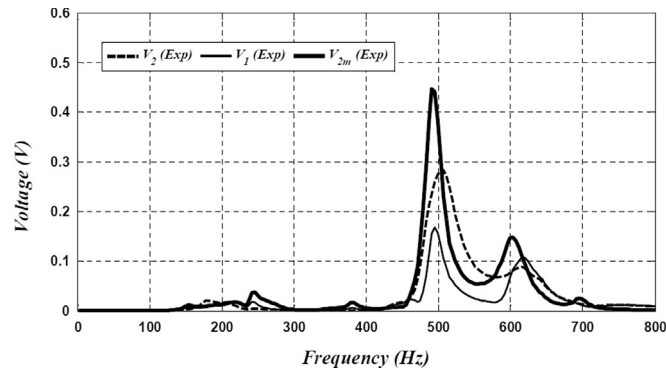


Fig. 6. Sine swept frequency response of voltage outputs V_1 , V_2 and V_{2m} of the two piezoelectric elements ($k_c = 29,180$ N/m).

side of the stack, and are thus slightly modified to reflect the speaker impedance at $x = 0$. The rest of the procedure, however, remains unchanged. The obtained theoretical characteristics and the corresponding experimental frequency responses are displayed in Fig. 7. The figure shows, to a great extent, a good agreement between theoretical predictions and experimental results.

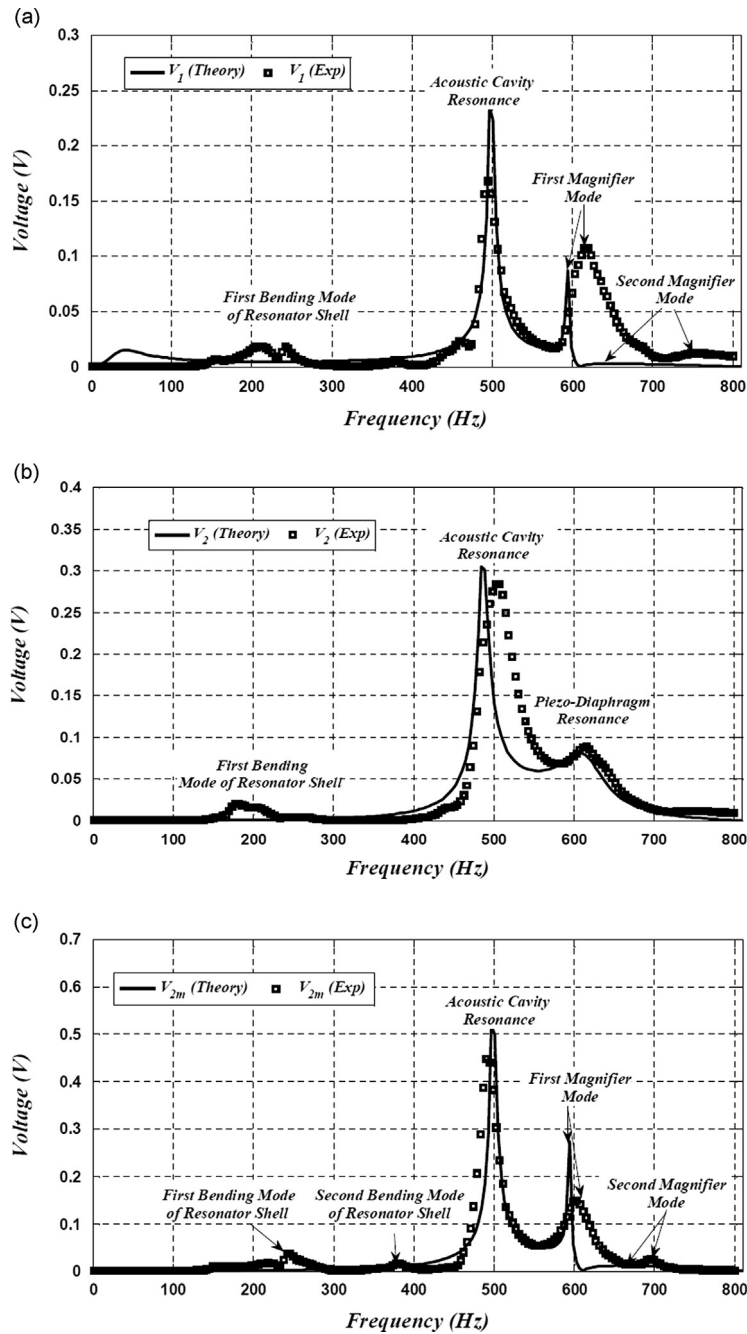


Fig. 7. Sine swept frequency response of voltage outputs (a) V_1 , (b) V_2 and (c) V_{2m} of the two piezoelectric elements ($k_c=29,180$ N/m).

Displayed on the figures are the identified structural and acoustic modes of the resonator as well as the structural modes of the piezoelectric diaphragm and magnifier system. Note that in the case of a harvester without a dynamic magnifier, the output voltage V_2 shows only a distinct peak at 480 Hz to indicate the acoustic resonance of the resonator cavity and another peak at 615 Hz to quantify the resonant frequency of the piezoelectric diaphragm. However, in the case of a harvester with a dynamic magnifier, the frequency spectrum of the output voltage V_{2m} shows two bending modes of the resonator shell at 230 and 380 Hz, acoustic resonance of the resonator cavity at 500 Hz, and combined resonant frequencies of the piezoelectric diaphragm and the magnifier at 605 Hz and 700 Hz, respectively.

3.1.2. Deflection of piezoelectric diaphragms

To verify that the voltage measurements presented earlier reflect the actual corresponding piezoelectric deflection and for the purpose of voltage-displacement calibration as well, the surface of the piezoelectric elements in the TAP and the

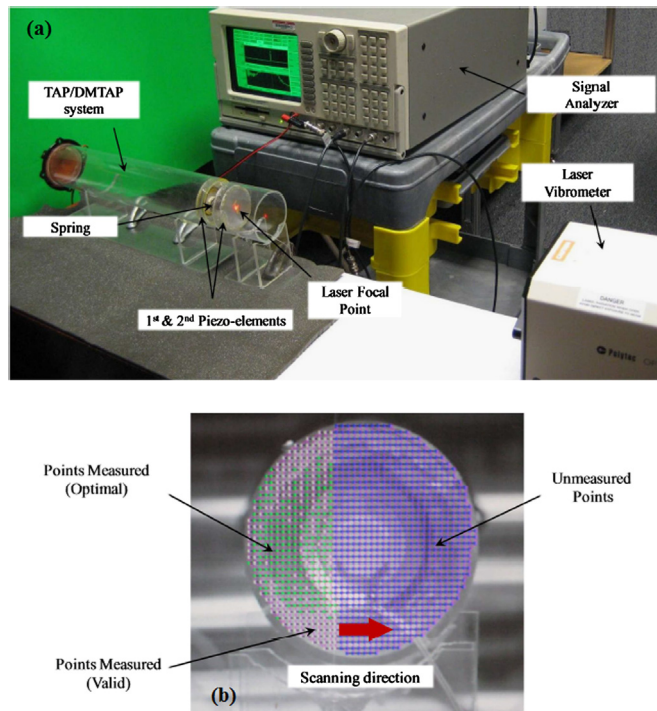


Fig. 8. (a) Experimental setup of the laser vibrometer and (b) the scanned surface of the piezoelectric elements to obtain values for the transverse deflection.

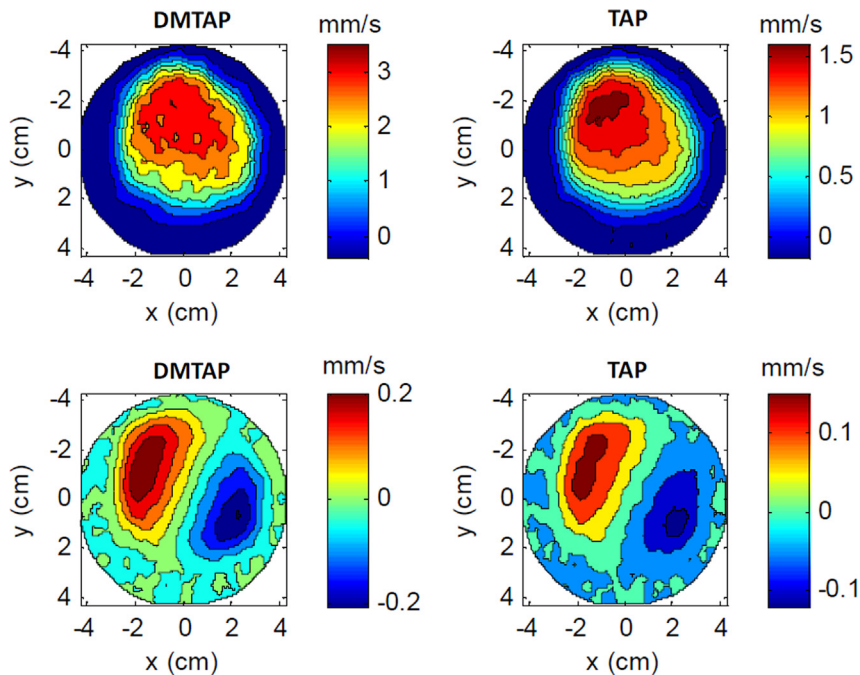


Fig. 9. Contours of transverse velocity of the DMTAP and the TAP showing the first and second deflection modes.

DMTAP cases is scanned using a laser vibrometer during operation. The contours are obtained using the PSV200 scanning Laser Doppler Vibrometer from Polytec-PI. The setup for this experiment is shown in Fig. 8(b).

Fig. 9 shows results from a vibrometer scan of the first piezoelectric element when the spring and the second piezoelectric element are detached (the TAP case) and a scanning of the second piezoelectric element after placing it back and attaching it to the first element using the spring (the DMTAP case). Contours of the transverse velocity clearly show the

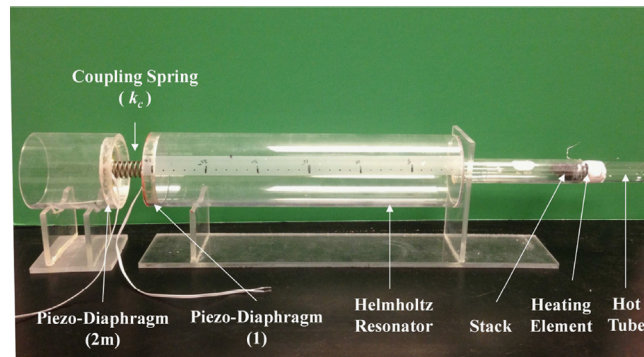


Fig. 10. Experimental prototype of a DMTAP harvester.

Table 1

Dimensions and gas properties of the experimental prototype of the TAP and the DMTAP harvesters.

Hot duct		
Length	L_1	6 cm
Diameter	D_t	1.95 cm
Stack		
Length	L_2	33.75 mm
Hydraulic radius	r_h	0.34 mm
Resonator		
Cold tube length	L_3	6 cm
Helmholtz resonator length	L_4	28 cm
Helmholtz diameter	D_R	7.1 cm
Piezoelectric diaphragm		
Stiffness	K_p	20,800 N/m
Capacitance	C_p	60e-9 F
Strain constant	d	-180e-12 m/V
Mass	m	0.0027 kg
Electric load	Z_L	1000 Ω
Working gas		
Gas type	Atmospheric air	
Isobaric to isochoric specific heat ratio	γ	1.4
Speed of sound in gas at mean temperature	c	469.03 m/s
Prandtl number	σ	0.631
Mean pressure	P_m	1 bar

difference in the deflection pattern between the first mode and the second mode. It is also evident that the measurements of the voltage output are confirmed as the DMTAP harvester is shown to have a significantly higher transverse velocity than the TAP in both modes. It is also shown that operating at the first mode is not only favorable to avoid having the piezoelectric element work against itself, but also because the piezoelectric elements experience much higher strain compared to the secondary modes. It is notable here that the voltage magnification ratio V_{2m}/V_2 found to be about 1.93 is in close agreement with the deflection magnification ratio x_{2m}/x_2 , which for the first mode shown in Fig. 9 is equal to 1.81.

3.2. Dynamically magnified thermoacoustic-piezoelectric harvester (DMTAP)

The performance of an experimental prototype of an actual DMTAP harvester with a stack is now investigated and compared with that of a conventional TAP. The experimental harvester is shown in Fig. 10, with the main dimensions and thermo-physical properties of the working gas listed in Table 1. The stack is heated from one side by means of a resistance heating wire and a DC power supply as shown in Fig. 10. The heating power input is kept constant at 40 W throughout all the carried out experiments to ensure that any increase in the harvested power is not a result of a higher thermal input.

The same concept is used whereby an identical piezoelectric element is used as the magnifier mass, such that by detaching the coupling spring the DMTAP harvester shown in Fig. 10 becomes a TAP for comparative purposes. Different springs with varying stiffness can be used to simulate different values of k_c , while small nuts with known masses can be placed at symmetric locations on the piezoelectric element to change the value of the magnifier mass m_m as shown in Fig. 11.

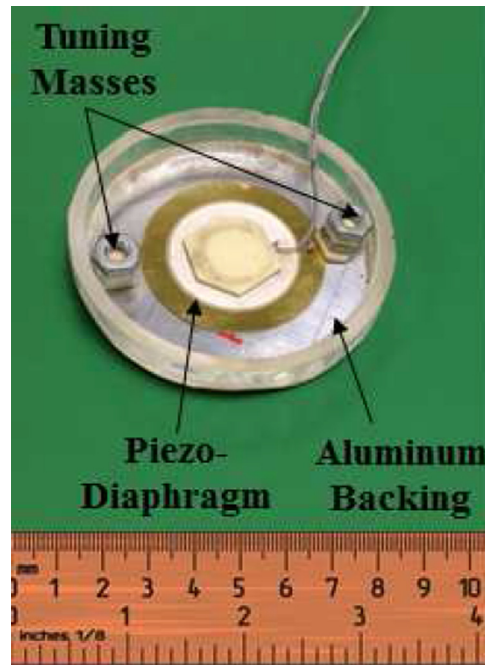


Fig. 11. Piezoelectric element on aluminum backing with tuning masses at symmetric locations.

Table 2
Springs and mass ratios used in the DMTAP experiments.

Experiment no.	Spring stiffness (k_c)		Stiffness of piezoelectric element (s)	k_c/s	m_m/m
	lb/in.	N/m	N/m		
(a)	33.15	5805.45	38,703.00	0.15	1.00
(b)	221.00	38,703.02	38,703.00	1.00	1.00
(c)	861.00	150,784.16	38,703.00	3.90	1.00
(d)	2.65	464.09	38,703.00	0.01	1.38
(e)	221.00	38,703.02	38,703.00	1.00	1.38
(f)	2.65	464.09	38,703.00	0.01	0.20
(g)	861.00	150,784.16	38,703.00	3.90	0.20

3.2.1. Voltage output from piezoelectric diaphragms

The stack-less speaker driven resonator in the previous section simulated the operation of a system similar to a DMTAP harvester and showed significant amplification of the harvested power from the piezoelectric element upon proper selection of the magnifier parameters. The speaker driven setup also enabled us to control the frequency of the acoustic oscillations and hence examine the harvested power over a swept range of frequencies. In an actual thermoacoustic-piezoelectric harvester with a stack and external heating, the self-excited oscillations should occur at the resonant frequency of the cavity coupled with the piezoelectric element or the dynamic magnifier system. As presented earlier, this was estimated to be in the vicinity of 400 Hz.

Experiments were carried out using different combinations of k_c and m_m , and for each the voltages across the two piezoelectric elements V_1 and V_{2m} were measured and eventually compared with the voltage V_2 from the piezoelectric element in the TAP case. Table 2 lists a selection of seven experiments (numbered 'a' to 'g') that were carried out with different stiffness ratios k_c/s varying from 0.01 to 3.90 and mass ratios m_m/m varying from 0.20 to 1.38.

Fig. 12 displays the peak to peak time oscillations of the voltages V_1 , V_2 and V_{2m} for each of the combinations listed in Table 2. The voltage magnification ratio V_{2m}/V_2 indicates how the voltage from the DMTAP harvester compares to that of the TAP and is of greatest interest to us. It can be seen that dynamic magnification does take place in plots (a), (b) and (e). Plots (c), (d), (f) and (g), however, show an undesirable decrease in the voltage from V_2 to V_{2m} . The best magnification ratio overall takes place in experiment no. (a), where a magnified peak voltage of 12.85 mV is generated from the piezo-diaphragm as opposed to 3.18 mV without the magnifier. This is achieved with a very small resistive load (close to open circuit conditions). The corresponding power output improves from 0.001 mW to about 0.0165 mW. The harvesting efficiency η_e , which is

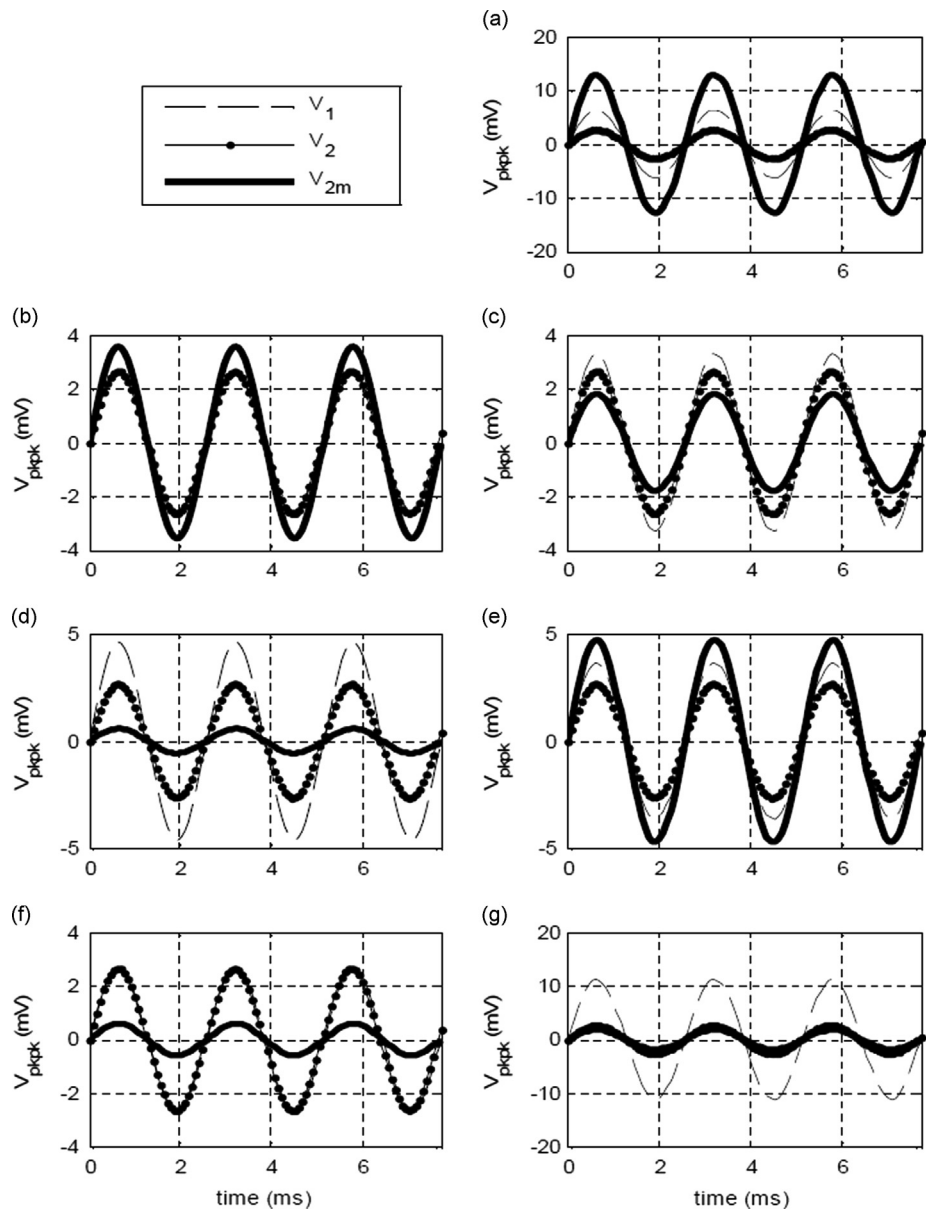


Fig. 12. Piezoelectric element peak-to-peak voltages V_1 , V_2 and V_{2m} for different configurations of springs and mass ratios given in Table 2.

Table 3
DMTAP to TAP voltage ratio: experimental results and theoretical predictions.

Experiment no.	Parameters		Experiment			Theory V_{2m}/V_2
	k_c/s	m_m/m	V_2 (mV)	V_{2m} (mV)	V_{2m}/V_2	
(a)	0.15	1.00	3.18	12.85	4.05	3.85
(b)	1.00	1.00	3.18	3.57	1.12	1.27
(c)	3.90	1.00	3.18	1.82	0.57	0.66
(d)	0.01	1.38	3.18	0.56	0.18	0.25
(e)	1.00	1.38	3.18	4.71	1.48	1.28
(f)	0.01	0.20	3.18	0.60	0.19	0.27
(g)	3.90	0.20	3.18	1.81	0.57	0.66

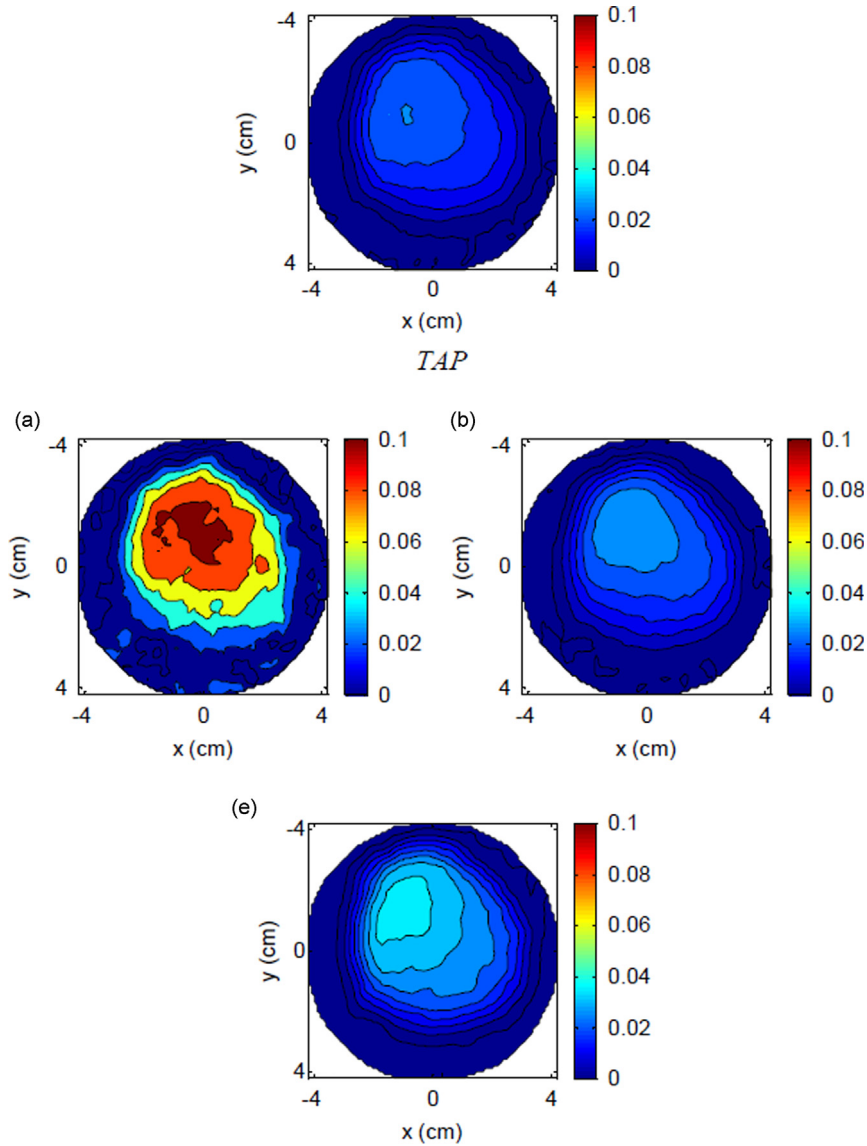


Fig. 13. Surface deflections of the piezoelectric element in the *TAP* experiments and the *DMTAP* experiments having a positive magnification ratio $V_{2m}/V_2 > 1$ (values in mm/s).

defined as the electric power output as a percentage of the acoustic power radiated to the end of the harvester [12], improves from 0.23 to about 2.1 percent. These values can be significantly enhanced by using optimal resistive loading as shown in [13].

A comparison between the predicted and the experimental values of V_{2m}/V_2 is presented in Table 3. The comparison reveals good agreement between the mathematical model and the experimental data. It should be noted here that the model presented earlier in Section 2 assumes a one-dimensional propagating wave in the x -direction. Hence, a piezoelectric element that is deflecting with one *dof*: x_2 in the *TAP* case and x_{2m} in the *DMTAP* case are considered. This is in contrast to a circular piezoelectric diaphragm fixed at the circumferential points. These limitations may have contributed to the minor discrepancies between some of the predictions and the experimental findings.

3.2.2. Deflection of piezoelectric diaphragms

Fig. 13 shows the scanned surface of the piezoelectric elements examined using the laser vibrometer for the *TAP* case, as well as the *DMTAP* experiments listed in Table 3 having a positive voltage magnification ratio (i.e. $V_{2m}/V_2 > 1$). The scans show the deflection in mm/s and are set to a fixed color bar scale to illustrate the amount of amplification that occurs in the deflection of the piezoelectric element when the *DMTAP* is adequately tuned. This is very clear in the vibrometer scan of the

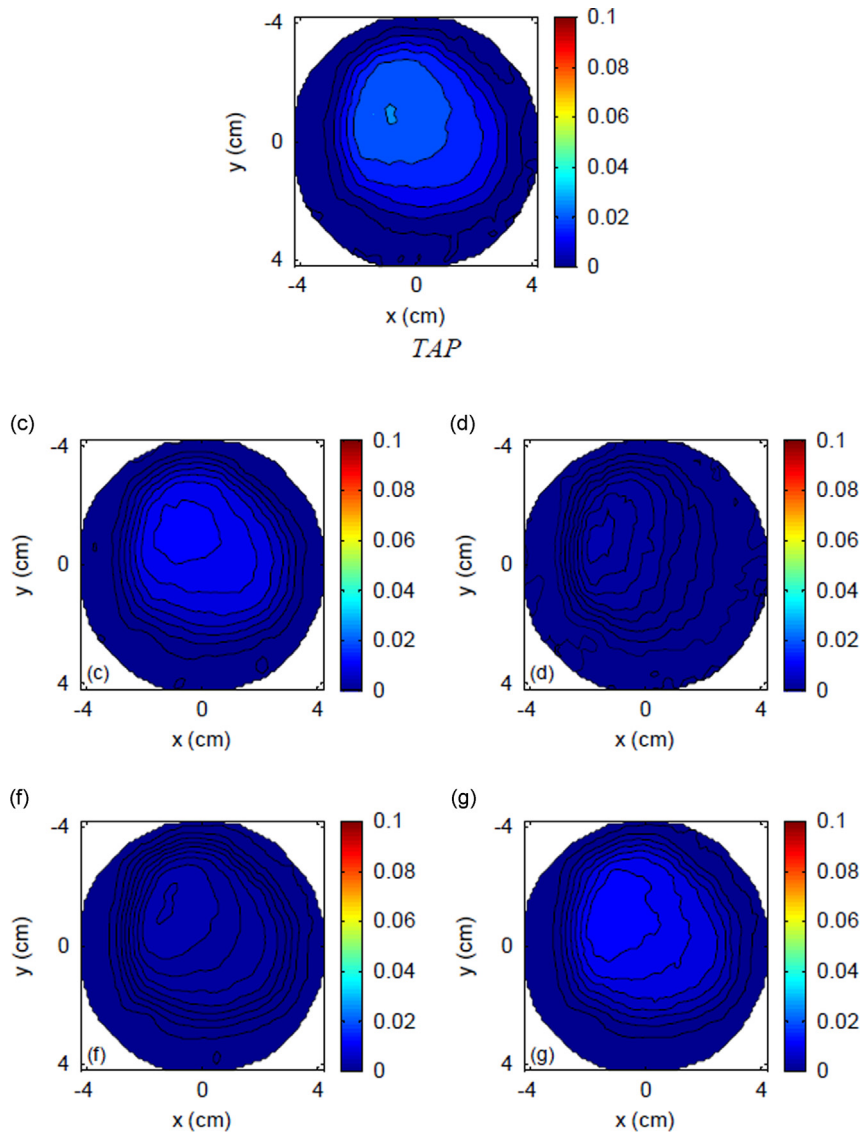


Fig. 14. Surface deflections of the piezoelectric element in the *TAP* experiments and the *DMTAP* experiments having a negative magnification ratio $V_{2m}/V_2 < 1$ (values in mm/s).

DMTAP used in experiment (a), where a deflection (and a voltage output) of about 4 times that of the conventional *TAP* is observed. Fig. 14 shows the rest of the *DMTAP* experiments, namely the ones where the selected magnifier parameters did not result in output voltage improvement. The *TAP* case is again placed for comparison. Finally, Fig. 15 shows the output voltage from the piezoelectric elements in relation to their deflection for all the carried out experiments.

4. Conclusions

This paper presented an experimental investigation of thermoacoustic-piezoelectric energy harvesters. The proposed enhancement to this class of harvesters was presented by augmenting the harvester with a simple spring–mass auxiliary system aimed at enhancing the strain experienced by the piezoelectric element placed at the end of the resonator. Tests were carried out on an experimental prototype of the harvesters with and without the dynamic magnifier.

The presented theory and the conducted experiments are limited to linear behavior of the thermoacoustic-piezoelectric energy harvester where low amplitudes of oscillations are adequate to develop a good understanding of the physical phenomena.

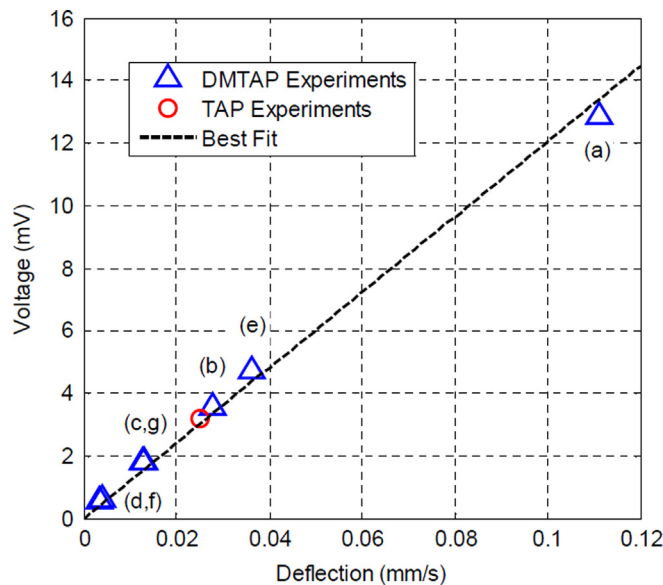


Fig. 15. Deflection against voltage of the piezoelectric elements for all the performed TAP and DMTAP experiments.

The carried out experiments demonstrated the superiority of the DMTAP over the conventional TAP energy harvesters in terms of both voltage output and piezoelectric movement. Equations developed for the TAP and the DMTAP in the earlier literature have been employed to verify the results and close agreement was observable. The obtained results demonstrate the potential of DMTAP systems as effective energy harvesters. The presented techniques can serve as invaluable tools to aid the design, buildup and further analysis of such prototypes.

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